

Answers for class prep quiz on section 4.4, Stewart's Calculus (8th ed.)

1. **Answer:** (c).  $\lim_{x \rightarrow \infty} \frac{h(x)}{g(x)}$  has the form  $\frac{\infty}{0}$ , and therefore, L'Hôpital's Rule does not apply. In contrast, (a), (b), and (d) have the form  $\frac{0}{0}$ ,  $\frac{+\infty}{+\infty}$ , and  $\frac{+\infty}{-\infty}$ , respectively, all of which are subject to L'Hôpital's Rule.

2. **Answer:** (a). We see that:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{4x^3 - x^2 + 7}{7x^3 + 14x - 13} &= \lim_{x \rightarrow \infty} \frac{12x^2 - 2x}{21x^2 + 14} && \text{(L'Hôpital)} \\ &= \lim_{x \rightarrow \infty} \frac{24x - 2}{42x} && \text{(L'Hôpital again)} \\ &= \lim_{x \rightarrow \infty} \frac{24}{42} && \text{(L'Hôpital again)} \\ &= \frac{24}{42} = \frac{4}{7}. \end{aligned}$$

Note that since

$$\lim_{x \rightarrow \infty} \frac{4x^3 - x^2 + 7}{7x^3 + 14x - 13} = \lim_{x \rightarrow \infty} \frac{4 - \frac{1}{x} + \frac{7}{x^3}}{7 + \frac{14}{x^2} - \frac{13}{x^3}} = \frac{4 - 0 + 0}{7 + 0 - 0} = \frac{4}{7},$$

we do not actually need L'Hôpital here; however, it does work.

3. **Answer:** (b). We see that:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\cos x - 1}{x \sin(3x)} &= \lim_{x \rightarrow \infty} \frac{-\sin x}{\sin(3x) + 3x \cos(3x)} && \text{(L'Hôpital)} \\ &= \lim_{x \rightarrow \infty} \frac{-\cos x}{3 \cos(3x) + 3 \cos(3x) - 9x \sin(3x)} && \text{(L'Hôpital again)} \\ &= \frac{-1}{3 + 3 + 0} = -\frac{1}{6}. \end{aligned}$$

Note that after one L'Hôpital, we still have the form  $\frac{0}{0}$ , but after two L'Hôpitals, we no longer have that form. Fortunately, at that point, the denominator is no longer 0 at  $x = 0$ , so we can evaluate the limit by plugging in.

4. **Answer:** (b). We see that:

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x}{3^x} &= \lim_{x \rightarrow \infty} \frac{1}{(\ln 3)3^x} && \text{(L'Hôpital)} \\ &= 0,\end{aligned}$$

because  $\frac{1}{(\ln 3)3^x}$  has the form  $\frac{1}{+\infty}$  as  $x \rightarrow \infty$ .